

# INITIATION AND GRADUAL EVOLUTION OF THE SCIENTIFIC SPIRIT OF THEODORE VON KÁRMÁN

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In the fall of 1956, following a persevering effort unimpaired by age, Theodore von Kármán gave birth to the idea of a world-wide cooperative endeavour in the realm of the theoretical and applied sciences which underlie aeronautics and its natural extension, astronautics.

The foundation of the International Council of the Aeronautical Sciences (ICAS) resulted from this, with the support of the most qualified scientific societies in this domain, representing today twenty-seven nations.

In calling upon me, thanks to the friendly confidence of the members of this Council, to preside over its Executive Committee, Theodore von Kármán called me also to a very close cooperation with himself and with the inner circle which had successively prepared the three preceding Congresses of ICAS held at Madrid (1958), Zürich (1960), and Stockholm (1962).

Everyone knows, or should know, the part played by the founder of ICAS in the success of these meetings, and it is appropriate here to re-emphasize it, and to express the gratitude of the entire Council.

The IVth Congress which opens today had held his active interest in equal measure. Another preparatory meeting had even been planned to take place during his brief visit to that historical city of Aachen where his exceptional career of professor and scientist had begun a half century earlier. There it happened that, several days before his birthday, during the night of May 6-7, 1963, he passed away in his sleep.

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Many are those who, accustomed to finding him at so many meetings in so many countries, expect to see him appear again among us, mount the rostrum and improvise one of those encouraging talks where he mixed thought and humour in his own inimitable style. Alas, only his shadow remains in our memory, and it is now from "that other side of the world"—which he sometimes mentioned wistfully—that he participates in our meeting.

Furthermore, on my proposal, the Executive Committee of ICAS decided to honour him today by dedicating to him the Daniel and Florence Guggenheim Memorial Lecture, which traditionally opens these Congresses, the first of which he had himself inaugurated in masterly fashion.

My initial intention was to try to sketch here an overall view of his contributions to the broad field of the aeronautical sciences in which he had laboured so fully. This naturally led me to read or review his collected works [1], which were compiled and published by his friends in 1956, and which pertain to the half-century 1902–1952. However, it very soon appeared impossible to condense in this lecture all the principal aspects of such a vast scientific production. Moreover, some excellent analyses have already been made, several by Dr. Hugh L. Dryden, distinguished by a very pertinent approach, based on comprehensive interpretations resulting from a long friendship. Also are to be mentioned here very recent analyses of this kind, namely by G. Temple, G. Gabrielli, E. T. Jones, and by F. Wattendorf, and F. Malina.

I had been directly familiar with the works in question only after 1925 on a sporadic but often intimate basis. My extensive and recent reading of the previous works—which had already extended over almost a quarter of a century—assumed for me a certain character of novelty. It immediately evoked within me, bridging the elapsed time, the illuminating revelations of the breadth of this incomparable scientific spirit which I had only known in its maturity. Fascinated by these revelations, it seemed to me that it would be more appropriate, especially because the man whom we honour today always appreciated originality in the subject of a lecture, and also more useful, to devote this talk to the "Initiation and Gradual Evolution of Von Kármán's Scientific Spirit," as it is revealed through his work from 1902 until twenty-five years later.

In truth, this subject, instructive in itself, appeared very appropriate to honour especially the memory of Theodore von Kármán's quality of being a "Great Teacher." This subject gives us, in fact, the rich lesson of his personal example with respect to this evolution of spirit which is at the same time a basic objective of all intellectual education, and a prime condition of its true value.

In 1902, at age twenty-one, the young von Kármán was graduated from the Royal Technical University of Budapest—the name it bore during the epoch of the Austro-Hungarian monarchy—and entered the employ of a reputed manufacturer of motors, the Ganz Company, as a research engineer, a rather remarkable qualification for that era and already indicative of his inclinations. During the same period, he published in the Mathematical Society of Budapest [2] a study of rational mechanics involving friction which concerned a little-known problem, that of a pendulum with spherical head rolling on a plane surface. He solved and discussed the problem completely; in this early work the author already demonstrated his talent for clarity of presentation and simplicity of mathematical expression, a talent he did not fail to cultivate in all his later publications. His continued development of this innate capacity, no doubt inherited from his father, a renowned professor of philosophy, enabled him thirty to forty years later, in very different surroundings, to transmit his knowledge without difficulty to his young American students in Pasadena.

In 1906, in the same spirit of clarification, and because problems of design encountered in industry had led him in this direction, he presented in Hungary another study of theoretical mechanics on the buckling of long, slender columns [3]. Here again appears his tendency to reduce a technical problem to its essential elements by means of skillfully chosen and carefully developed simplification. In order to become master of this art, von Kármán knew that he must learn to become familiar with the complex conditions of reality. Already in this period of close contact with industrial problems, his progress in this respect is revealed by his efforts to sift, analyze and categorize the best experimental data and methods.

At this point the Hungarian period of our young graduate comes to an end. He moved to Germany, to the renowned University of Göttingen, to pursue higher studies simultaneously in mathematics, physics, and mechanics to obtain, in 1909, his Doctor of Science degree, and to remain there four more years as assistant professor.

There, probably because he followed the hereditary inclination already mentioned, he oriented himself toward teaching, and concentrated on mechanics. He prepared himself with the greatest care to combine theory with experiment, abstraction with reality, science with technology. He was already convinced that, contrary to an opinion unfortunately still flourishing in certain quarters, *mechanics is in itself a fundamental discipline*, distinct from mathematics as well as from general physics, although its principles are of physical origin and its rigor is based on pertinent application of mathematics, to which mechanics itself has moreover brought numerous helpful contributions.

Von Kármán was by then equally convinced that the application of this fundamental discipline justified approximations, avoiding at times the need for research of excessive precision and permitting the establishing of an attractive theoretical framework, based on simplifying assumptions which, although seemingly imperfect to the eyes of the pure logician, are compatible with preponderant principles considered essential.

It is easy to recognize and understand from his works of that period that the stay of our Hungarian student and engineer, soon to be Doctor, among the eminent masters of the University of Göttingen—whose memory he frequently recalled, notably Ludwig Prandtl—had strongly contributed to his personal development. In this milieu, at the same time studious, limited in dimensions and fruitful in associations, he was able to appreciate the full importance of deep study of fundamental questions, requiring continuous and concentrated effort.

Göttingen was then an intellectual center—some would perhaps now say a “summit” of serious teaching, profound culture, and advanced research. There the natural gifts of von Kármán became expanded and powerfully strengthened. In spite of the evident differences in their temperaments, the molding influence of that great master of theoretical and applied mechanics, Ludwig Prandtl, manifested itself in a very telling manner on his student at that time. It is thus, notably, that the observation of phenomena prior to attempting their interpretation was impressed upon him as a “golden rule” from which he never deviated. His succeeding works of this period well illustrate this.

In 1907 and 1908, for example, he published two studies, one on stationary waves in gas streams [4] and the other on the flow of vapors and gas in nozzles [5]. This first entry of his into the domain of supersonic flow, shock waves, and detonation waves gave advance notice of his second masterly incursion into this field, namely, at the Volta Congress in 1935. However, this also followed two studies of Prandtl, published in 1905 and 1906, in the *Physikalische Zeitschrift*, and to which he had obviously given profound attention. Thirty years later this seed planted in his spirit was to come into full flower.

However, after the first entry into an advanced sector of fluid dynamics the activities of Theodore von Kármán were to remain for a long time devoted to an entirely different aspect of continuum mechanics, that of deformable solids, without ever losing sight of the very technical problems associated with his first contacts with the motor industry in Hungary. In this respect, mention should be made—especially because of its title which reveals his attention to progress—of a study published in Budapest, entitled: “Very Lightweight Engines” [6], inspired by his visit to the Paris Aviation Salon of 1908, during which he exercised his talents for

observation and reflection. In this long-ago period a designer visualized reducing the specific weight of a motor of 120 hp, then considered as "high power," to about 1 kg per hp by increasing the number of cylinders from 8 to 16!

Thus, concentrating his work on the mechanics of deformable solids, von Kármán attacked problems that went beyond the classic linear elasticity of Hooke concerning the definition of the "elastic limit" [7] or, in collaboration with A. Haar, the theory of constraints in plastic or granular media [8]. In this latter study, he traces concepts which particularly illustrate his effort to distinguish the physical aspects of a problem from its mathematical interpretation. Here he discussed the ideas of pioneers such as Saint-Venant, Rankine, M. Lévy, dating back over forty years, whose works he had assiduously studied. Today, after the passage of another forty years, these studies of von Kármán still remain up to date, and his conclusions still warrant useful consideration. Thus his example teaches us that, in problems which are very difficult because of their physical complexity, scientific thought progresses only very slowly, and that one should never neglect the heritage of the thinkers who have preceded us.

In 1909 von Kármán defended before the Faculty of Higher Philosophy of the University of Göttingen, his Doctor of Science thesis. This dealt with "Studies on Resistance to Lateral Flexion." Having chosen slender columns as the subject of his research, the author presented a considerable number of experiments conceived, carried out, and analyzed in systematic fashion. The detailed and logically systematic conduct of this work, based on experimental observation, show conclusively the mastery acquired by von Kármán in the deep knowledge and advanced research so characteristic of the Göttingen School at that period. He himself called attention to the contributions made by his master L. Prandtl to this effort, in his capacity as Faculty Advisor. The imprint of this rich, fruitful, and realistic training was evident in all of the succeeding scientific work of the new Doctor of Mechanics who, at only twenty-eight years of age, already possessed extensive experience.

Soon afterwards, benefitting from his new occupation as assistant professor at Göttingen, von Kármán inaugurated his career of teaching by producing two works of a didactic character, the second in collaboration with his colleague and friend, L. Föppl. These works are worth citing as examples. The first constituted a condensed but very complete and remarkably well-organized review of "Problems of Resistance in Machine Design" [9]. It is a coordinated synthesis of methods and formulas, extracted from original works of more than forty great masters of Mechanics whose activity extended from 1850 to 1910, and this synthesis succeeded in producing a widespread unity built on the basis of diversity.

The second work treats of the “physical” fundamentals of teaching resistance of materials. [10] Here again the comparison and discussion of previously expressed ideas are carried through with order and with clarity, in referring to lessons learned from well-carried-out experimentation. It is here particularly that one sees in the thoughts of the author, perhaps because of his association at the time with E. Föppl, the clear distinction between physical irreversibility and mathematical nonlinearity, which do not necessarily go together.

The foregoing exposé gives us a clear insight into the evolution of the scientific way of thinking of von Kármán, who with regard to those very difficult problems—which today are still only very incompletely solved—extends his scope and explores new pathways. In the analysis and discussion of concepts of retarded deformation, relaxation, hysteresis, flux, he recognized their complexity, referring to the ideas presented by Duhem in 1894–1898 as well as to the concepts of J. C. Maxwell (1890) and of Boltzmann. It is impossible here to describe or even sketch in a few words the immense panorama of research reviewed and mentioned under 175 separate and distinct references. The conclusion of this work—the reading of which cannot be too highly recommended—is truly remarkable for that era. It expresses the necessity of associating thermodynamics, elasticity, magnetism and chemistry with mechanics in order to understand rather completely how the strains in deformable solids are related to their interior state and to the constraints exercised on them.

This is the basic thought which for twenty years has been guiding a great number of mechanicians throughout the world, concerning the evolution of the mechanics of the continuum, that is to say, of fluids as well as solids.

In passing, let us underline that the two exposés just discussed illustrate magnificently the intelligent concept of the “Scientific German Encyclopedias” of that time. Published by competent and conscientious editors, they were continually brought up to date by calling on young scientists, whose personal review of literature they enhanced while leading them on toward producing homogeneous didactic presentations of high quality.

The preceding characteristics of the works of von Kármán at Göttingen are likewise confirmed in two other papers published in 1911 in the famous *V D I Journal*, where detailed experiences confirm the concept of the elastic limit according to Mohr in the case of very different materials, in “compression” under uniform external pressure. However, the author prudently admits the necessity for correlative verification under uniform “depression.”

In spite of his attachment to mechanics of solids, Theodore von Kármán felt himself at times attracted to the mechanics of fluids which he had

never forgotten. An article by E. Bose and his wife, published in 1911 in the *Physikalische Zeitschrift*, gave him the occasion for a fundamental statement based on dimensional analysis concerning the still controversial notion of turbulent viscosity, a subject on which he was to produce essential works between 1920 and 1930.

Later, in 1911 and 1912, von Kármán presented to the Scientific Society of Göttingen a paper in two parts on the "Hydrodynamic Resistance" [11]. Rejecting the concept of stationary wakes and dead water, he treated the two-dimensional problem, making use of various experiments including those of Benard in France three years earlier. He developed the concept of a wake consisting of a series of vortices in a parallel and alternating pattern, which later became known as the "Kármán Vortex Street." He applied himself also to the boundary layer theory of Prandtl, and established, by considering the stability of the vortex pattern, a first formula of the relationship between the width and the spacing of this vortex pattern. In fact he committed an error of estimation—an excessively rare occurrence in the publications of von Kármán. Shortly afterwards, he published in collaboration with H. Rubach [12] a paper in which he resumed the study of the double pattern of vortices, in which the width-spacing relationship is reduced from 0.359 to 0.283, a figure later confirmed by experimentation.

At this time, von Kármán left Göttingen for the University of Aachen to teach while founding and directing the new Aeronautical Institute, which was to encompass all research having to do with aeronautics. This concept is one to which ICAS itself is also devoted.

However, in his personal research von Kármán extended still further his efforts in the field of solid mechanics, orienting them at this time towards theoretical approaches even at microscopic scales, whose importance and illustrative value he knew.

It is notable that he published with Max Born, in 1912 and 1913, three important papers [13–15] on the vibrations of a molecular network, on the theory of specific heats, and on molecular flow in porous media, respectively. In some specialized fields these publications caused developments which we cannot consider here. However, they afforded the principal author a deep insight into the kinetic gas theory, into the reticulated structure of crystals, into Knudsen's research on rarefied gases, and finally into mathematical methods of statistical physics. During these two years which brought his previous scientific training in Göttingen to full fruition, Theodore von Kármán increased his knowledge and his personal insight into several areas for which he correctly predicted an important future.

Such extensive scientific development of a young professor of thirty-two with well-established roots in a diversity of fruitful discipline, makes it easy to understand how, at more than seventy years of age, von Kármán

could without difficulty start to work on the dynamics of rarefied gases, having already considered the deep significance of the "Knudsen Number" forty years earlier. This explains also that, at seventy-five, he was able to give to young admirers a general appreciation of magneto fluid dynamics which was a revelation to them in view of their concentration on partial and apparently new results.

Again, in 1913, he contributed four chapters [16] to one of the German encyclopedias mentioned above concerning the elasticity of solids, their resistance properties, their mechanical equilibrium and their endurance characteristics respectively. The last of these topics was originated by von Kármán. These four chapters were new cornerstones in the treatment of solid mechanics, which he progressively pioneered while at the same time teaching and directing research. Due to the simplicity and clarity of exposition, which he had by now mastered and used with artistic skill, these general reviews of very complex problems were remarkably concise. In about one hundred pages he managed to condense the substance of a work which normally would have been twice or three times as long. This clarity, simplicity, and conciseness, while attributable to an innate talent, were certainly enhanced by the experience acquired in seven years of intense personal work.

Little by little the general aeronautical research goals of the new Institute at Aachen, which he directed, attracted von Kármán toward problems of the study of which a new impetus was given by his earlier bent toward technical fruition.

As a result, in 1914, the *Jahrbuch der Wissenschaftlichen Gesellschaft für Luftfahrt*—a review of considerable importance—published his paper, written in collaboration with E. Trefftz, on "Stability and Longitudinal Oscillations in Airplanes." In this paper the principal author affirms his wish to establish in this field, which had already been treated in different approaches, a "golden road" to be followed forming a mean between analytical calculations and the purely mechanical aspect.

Then and now, this study established such revolutionary concepts as that not definitive indeed, of the "metacentric curve." I cannot resist the temptation to reproduce here the second figure of his text (Fig. 1), because it is significantly prophetic of the monoplane with swept back tail assembly, characteristic of today's modern airplane.

After World War I, which von Kármán spent on active duty with the Austro-Hungarian Army and during which he initiated the development and test of an helicopter of his own design, he returned to the Institute at Aachen. Turning away progressively from the study of solid mechanics and the behaviour of materials which had monopolized his attention for several years, he became more and more preoccupied with fluid mechanics, par-



ticularly as applied to experimental aerodynamics. He not only extended his early work at Göttingen but even succeeded in finding new directions.

Again, with Trefftz [17] he published in 1918 a paper on the theory of profiles based on conformal transformation. This application of the theory of analytical functions, touched upon a century earlier by Cauchy, became classical ten years later, but here von Kármán and Trefftz were forerunners. This study of the two-dimensional harmonic potential preceded a long and exhaustive series of contributions to the theory of lifting wings. The first two of these [18,19] date back to 1929, the second of which was in collaboration with K. Friedrichs. These contributions led to important extensions much later in the United States, fitting with remarkable ease into the domain of supersonic flight.

However, the major part of Theodore von Kármán's personal research at Aachen was devoted to skin friction, turbulence, and to the problems of boundary layer in its laminar, mixed flow, and turbulent aspects. Here our author was able to utilize fully the teaching of Prandtl and his own elaborations thereof, in the fields of statistical mechanics, dimensional analysis, and critical observation of physical phenomena of contact between flowing liquids and solid walls.

Two of his fundamental ideas in this field appear in his first publication of 1921 [20], greatly condensed in their mathematical expression, and reduced to two dimensions for maximum simplification. Certainly it was the

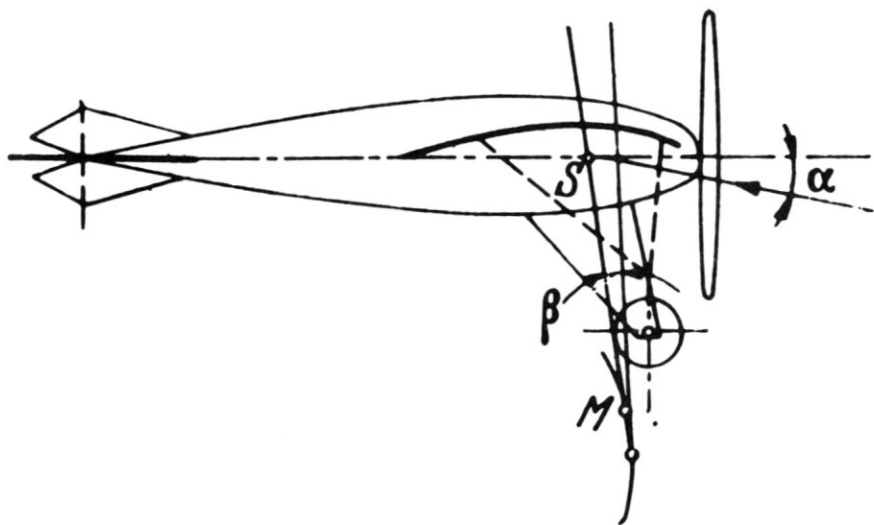


Figure 1.

result of long and patient previous meditation. The first idea is the treatment of intrinsic similitude, the second one distinguishes between the close proximity of a wall and the remainder of a turbulent boundary layer. Von Kármán was to explain this conventional and convenient distinction on many subsequent occasions but he was fully aware of its imperfection to consider it a rigorous theory. He expressed this in his whimsical fashion, saying that one beauty spot does not ruin a pretty face. It should be mentioned also, as proof of his judgment in the choice of valid references, that von Kármán cites, at the outset of his work on turbulence, the works of Boussinesq which date back to 1877 and which he regarded very highly, in contrast to many of his contemporaries.

During this particular era (1921) and in this field, von Kármán advanced his own concepts and formulated some bold hypotheses as, for instance, when he transposed the boundary layer of a tube to that of a flat plate. However, he would always confirm an audacious hypothesis with experimental proof, which illustrates the rigid standards which he had established for his scientific thinking.

At the same time he became extraordinarily adept at handling the processes of transformation and resolution, both rigorous and approximate, of the differential and integral equations resulting from the mathematical formulation of his problems. This was based on his ability to assimilate the most recent advances in mathematics, in view of his exceptional grasp of their possibilities and conditions of usefulness. In this connection we might mention in passing that von Kármán was to the letter a Master of "Applied Mathematics."

In his first basic paper [20] of 1921 on laminar and turbulent friction, it is evident that von Kármán's thoughts were still moving in Prandtl's wake, since a closing remark mentions that Prandtl had previously told him of his arrival at an identical conclusion, using a different approach. The difference in temperament and in thought processes between these two scientists, the older of whom had exerted such profound influence over his disciple, explains how in several instances they arrived at identical answers while using different methods.

An equally fundamental paper [21] shows, three years later, in 1924, that von Kármán's concept of turbulence had become more personal and all-inclusive. Specifically, this paper expands upon a basic distinction between thermal and turbulent agitation, symbolizing the microscopic and macroscopic aspects respectively. He emphasized this distinction by presenting the Reynolds number as the natural product of two dimensionless factors, representing these two aspects.

To conclude this brief review of a period during which von Kármán's scientific thought formed and developed towards complete mastery of the

entire field of mechanics, I should like to cite two more papers which are particularly indicative of his general philosophy in this area.

The first is a paper [22] which he presented in 1928 at the Mathematics Congress at Bologna, on "Mathematical Problems of Modern Aerodynamics." In this work he presents an inventory of resolved versus unresolved problems. "Modern" aerodynamics in this case was, of course, that of the period since the quality of modernity is lost with the passage of time. However, the list of unresolved problems established in 1928 retains its topical freshness in many respects, and can still serve as an inspiration for today's mathematicians.

During his entire career von Kármán emphasized periodically the unresolved problems in the various branches of the field of mechanics by reviewing them at Congresses or similar meetings. This was a remarkable feat of effective, long-range teaching.

The second example [23] is a presentation he made on "Mathematics and the Technical Sciences" at Göttingen in 1929 during the inauguration of the Mathematics Institute belonging to the University where he had, in fact, received his own mathematical training. There he gave an historical and very personal review of a subject which had always stimulated and occupied his curiosity. There he also presented some radically new concepts, among them the relationship between the "kernel" and the "influence" functions of an integral equation, and the suitability of demonstrating or admitting, depending upon the nature of a problem, the convergence of an expansion. His conclusion, which foresaw the future of applied mathematics, ensured for it through fidelity of contact, preservation of the fundamental principles which assure scientific unity in its so-called pure forms as well as its applied forms.

We have reviewed at some length but still in a schematic fashion some twenty-five years in the life of the founder of the ICAS in his capacity of scientist, teacher, and researcher. Apart from the many personal contributions made by him to the progress of mechanics in all its aspects, particularly in the aeronautical sciences, which his students brought to further fruition as in a chain reaction and which will continue for a long time to come, this review of the evolution of von Kármán's scientific spirit leaves us a lesson which is perhaps the greatest of his achievements—an example to follow.

Truly, his example shows us that the strength and importance of his work were based upon conscientious and extensive training and on extremely methodical and persevering work, guided and served by an exceptional intelligence.

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